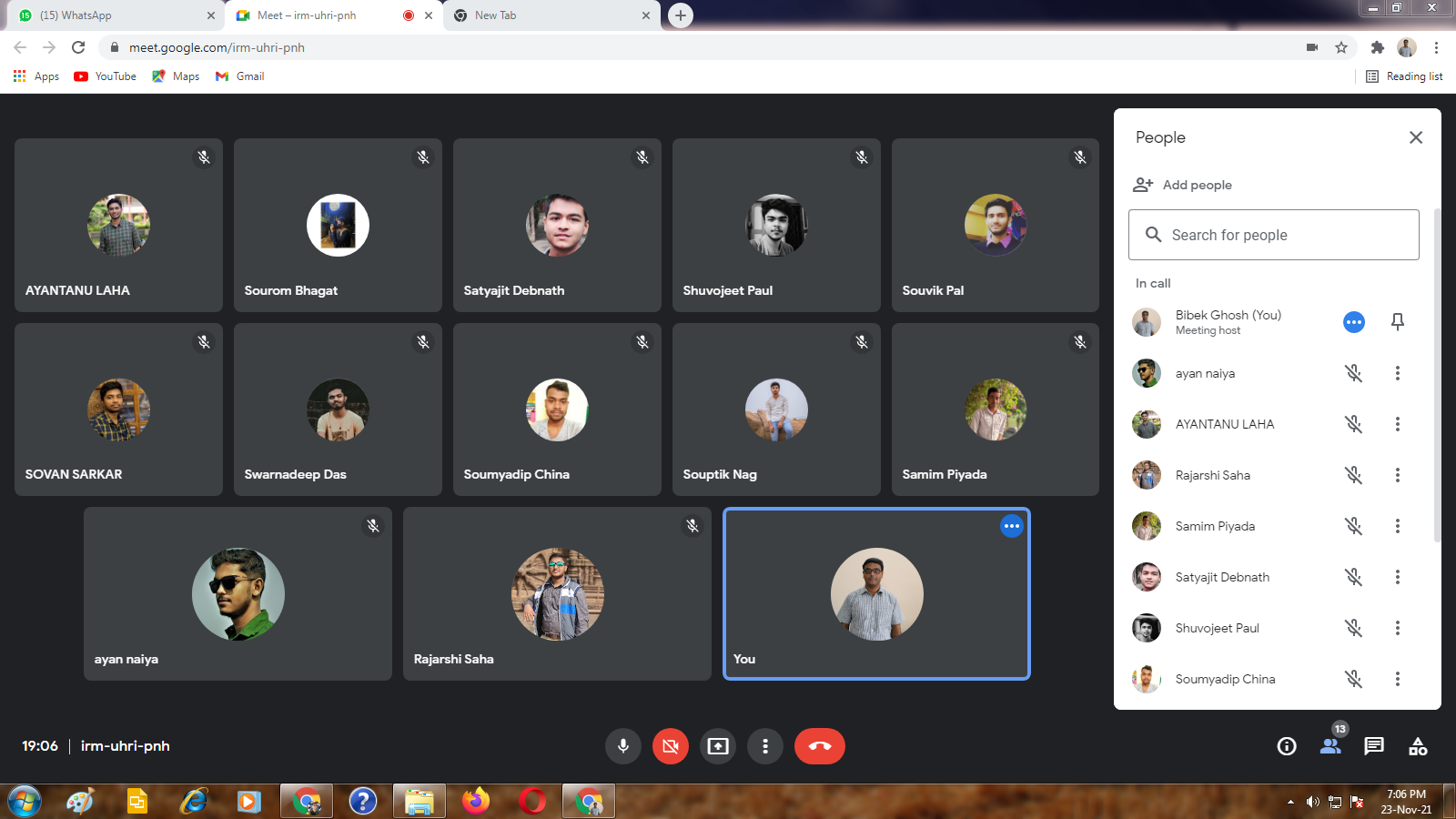
**Class 23/11/2021**

**Graph Theory**

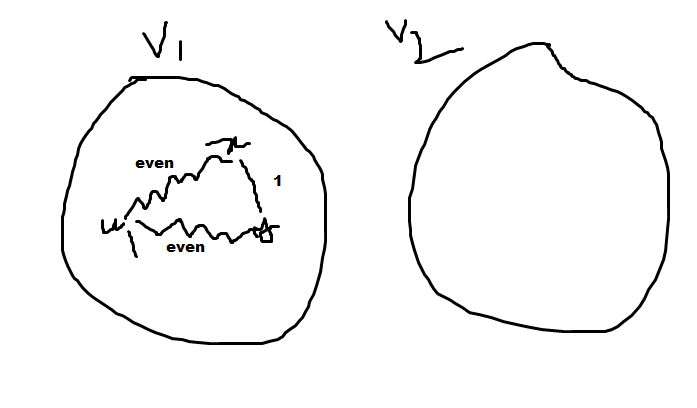


**Theorem: Prove that a graph is bipartite if and only if all its cycles are of even length.**

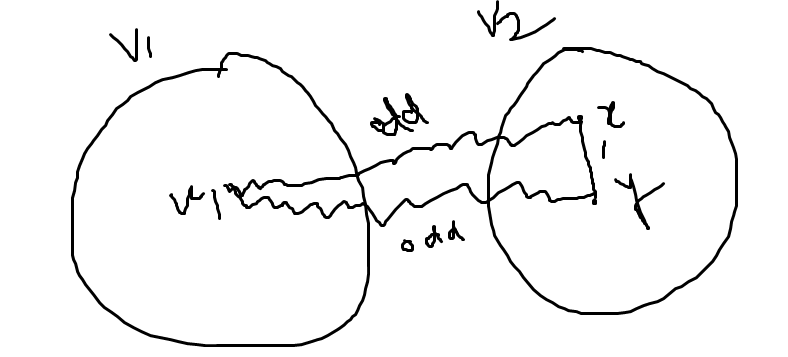
**Proof:** Let us consider a bipartite graph G (V,E). So its vertex set can be partitioned into two disjoint subsets say V1, V2 where V1= and V1. Let us consider a cycle C. Now for the sake of generality we consider u1 then u2u3V1, u4 V2 and so on. Hence oddly subscripted vertices belongs to V1 and evenly subscripted vertices belongs to V2. As and u1 are adjacent. Hence . Hence m must be even. So we can conclude a bipartite graph has all its cycles are of even length.

To prove the converse, we consider a graph G(V,E) whose cycles are of even length. We are taking an arbitrary vertex u1 and we are creating a set of vertices V1 which contain all vertices which are at even length distant from u1. And also we include u1 in V1. We create another vertex set V2=V-V1. So V1= and V1. Now we claim that our partition is bipartite.

Let us consider two vertices say x, y and let them be adjacent. Now we can create a cycle C1= {u1,…x,y,….u1}



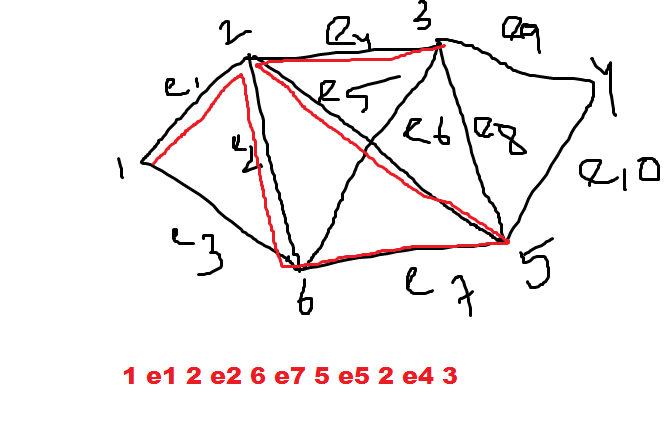
Now length of u1 to x=even and length of u1 to y=even. So if x, y are adjacent we can say that length of cycle C1 is odd. But it is a contradiction as all cycles of G are of even length. Hence x, y can’t be adjacent. So for any pair of vertex in V1 is not adjacent.



Let us consider two adjacent vertices x,y . Now length of u1 to x is odd else x must belong to V1.Similarly length of u1 to y is also odd. Now the cycle C2={u1..x,y…u1} must be of even length. So x and y must not be adjacent. No two vertices of V2 is adjacent.

Hence the graph G is bi-partite. (Proved).

**Walk:** A finite alternating sequence of vertices and edges beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it.

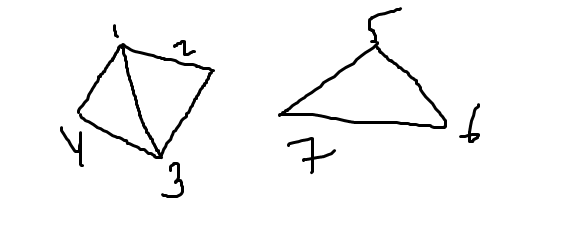


**Closed walk**: A walk whose beginning and end vertices are same is called closed walk.

**Path:** An open walk in which no vertex appears more than once is called a path. The number of edges in a path is called length of the path.

**Closed path or circuit:** A closed walk in which no vertex appears more than once except the beginning or end vertex.

**Connected Graph:** A graph G (V, E) is connected, if there exist at least one path between any pair of vertices of V. If not the graph is **disconnected**.



Disconnected graph G

A disconnected graph has many connected components.

**Theorem: A simple graph with n vertices and k components can have at most edges.**